LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2025**

PMT1MC02 – REAL ANALYSIS-I

Date: 25-04-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

	SECTION A – K1 (CO1)	
	Answer ALL the questions	**(5 x 1 = 5)**
1	**Answer the following**	
a)	Define metric space.	
b)	If $h(x) = sin\frac{1}{x}$, $\forall\, x \neq 0$ in $\mathcal{R}$, find $h'(x)$.	
c)	What distinguishes the Riemann Stieltjes integral from Riemann integral?	
d)	Define uniform convergence of a series of functions.	
e)	Why is the Weierstrass approximation theorem important?	
	SECTION A – K2 (CO1)	
	Answer ALL the questions	**(5 x 1 = 5)**
2	**MCQ**	
a)	Let (M, ρ) be a metric space and $a \in M$ then the set $\{x \in$M: $\rho(x, a) \leq r\}$ is called……. (i) open Set (ii) closed Set (iii) closed Sphere (iv) open Sphere	
b)	If f has a derivative at c then it is ………..at c. (i) continuous (ii) bounded (iii) closed (iv) none of the above.	
c)	If P is refinement of Q then (i) $L(P, f, \propto) \geq L(Q, f, \propto)$ (ii) $L(P, f, \propto) \leq L(Q, f, \propto)$ (iii) U$(P, f, \propto) = L(Q, f, \propto)$ (iv) U$(P, f, \propto) \geq U(Q, f, \propto)$	
d)	If $\{f_n\}$ is a sequence of continuous function on E and if $f_n \to f$ uniformly then f is ……… on E. (i) continuous (ii) discontinuous (iii) closed (iv) differentiable	
e)	Every equicontinuous family on a ……. Is uniformly bounded (i) closed set (ii) derived set (iii) compact set (iv) none of the above	

	SECTION B – K3 (CO2)
	Answer any THREE of the following **(3 x 10 = 30)**
3	Show that continuous image of a connected set is connected.
4	If f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$.defined on $[a, b]$. Show that there is a point $x \epsilon (a, b)$ such that $f'(x) = \lambda$.
5	If $f_1, f_2 \in \mathcal{R}(\alpha)$ on $[a, b]$. Show that $\int_a^b (f_1 + f_2) d\alpha = \int_a^b f_1 d\alpha + \int_a^b f_2 d\alpha$.
6	Let E be a subset of $\mathcal{R}$, suppose $\lim_{n\to\infty} f_n(x) = f(x)$ pointwise,$x \in E$ and $M_n = \sup_{x\in E} \lvert f_n(x) - f(x) \rvert$ then explain that $f_n(x) \to f$ uniformly iff $M_n \to 0$ as $n \to \infty$.
7	Present and prepare that there exists a real continuous function on the real line which is nowhere differentiable.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	Suppose f is a continuous mapping of a compact metric space X into a metric space Y. Then criticize that $f(x)$ is compact.
9	State and prove generalized mean value theorem.
10	a) Analyze that the lower Riemann-Stieltjes integral cannot exceed the upper Riemann-Stieltjes integral. b) Let f be a bounded real valued function defined on $[a, b]$ and α be a monotonically increasing function on $[a, b]$. if $f(x) = x$ and $\alpha(x) = x^2$. Does $\int_0^1 f \, d\alpha$ exists? If it exists, find its value. **(8+4.5)**
11	Let α be monotonically increasing function on $[a, b]$ and let $\{f_n\}$ be a sequence of real valued functions defined on $[a, b]$. Such that $f_n \in RS(\alpha)$ on $[a, b]$ for $n = 1,2,3$ ...If $f_n \to f$ uniformly on $[a, b]$, Then determine that f is itself integrable and $\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	a) Suppose f and g are defined on $[a, b]$ and are differentiable at a point $x \in [a, b]$ then $f + g$, $f.g$, f/g are differentiable at x, then support that (i) $(f + g)'(x) = f'(x) + g'(x)$ (ii) $(f.g)'(x) = f'(x)g(x) + g'(x)f(x)$ (iii) $(\frac{f}{g})'(x) = \frac{g(x)f'(x) - g'(x)f(x)}{(g(x))^2}; g(x) \neq 0$ b) Let $f(x) = \begin{cases} x^2, & x \neq 1 \\ 0, & x = 1 \end{cases}$ determine that $\lim_{x\to 1} x^2$ if limit exists. **(12+3)**
13	a) State and prove Cauchy criterion for uniform convergence. b) Test the uniform convergence of the sequence of the function $f_n(x) = x^n$ on $[0, k]$ where $k < 1$. **(12+3)**
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	a) State and demonstrate the necessary and sufficient condition for $f \in \mathcal{R}(\alpha)$ b) Suppose $\{f_n\}$ is a sequence of functions on E and suppose $\lvert f_n(x) \rvert \leq M_n$, $x \in E$ then $\sum f_n$ converges uniformly if $\sum M_n$ converges. **(12+8)**
15	Discuss and justify whether a uniformly continuous polynomial P_n is real for a continuous complex function f in $[a, b]$.

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